書目計量學

Lecture 02 -- 基本數學

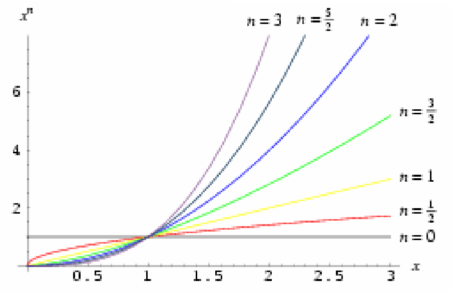
陳光華
國立台灣大學圖書資訊學系
國立台灣師範大學社會教育學系
khchen@ntu.edu.tw

Declaration

- The content of this slide is extracted from MathWorld (http://mathworld.wolfram.com)

Exponent

- An exponent is the power n in an expression of the form x^n.



Exponent Laws

$x^m \cdot x^n$	$=$	x^{m+n}	$\left(\frac{x}{y}\right)^n$	$=$	$\frac{x^n}{y^n}$
$\frac{x^m}{x^n}$	$=$	x^{m-n}	x^{-n}	$=$	$\frac{1}{x^n}$
$(x^m)^n$	$=$	x^{mn}	$\left(\frac{x}{y}\right)^{-n}$	$=$	$\left(\frac{y}{x}\right)^n$
$(xy)^m$	$=$	$x^m y^m$			

Logarithm

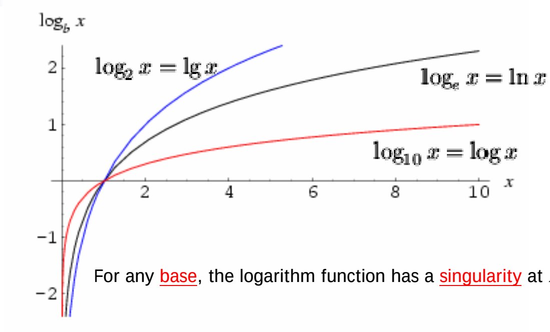
The logarithm $\log_b x$ for a base b and a number x is defined to be the inverse function of taking b to the power x, i.e., b^x. Therefore, for any x and b,

$$x = \log_b(b^x),$$

or equivalently,

$$x = b^{\log_b x}.$$

Logarithm Curve



For any base, the logarithm function has a singularity at $x = 0$.

Source: http://mathworld.wolfram.com/Logarithm.html

Some Properties of Logarithm

$$xy = b^{\log_b x} b^{\log_b y} = b^{\log_b x + \log_b y}$$

$$\log_b x + \log_b y = \log_b xy$$

$$\log_b x - \log_b y = \log_b (x / y)$$

$$n \log_b x = \log_b x^n$$

Some Properties of Logarithm (continued)

$$\begin{aligned} a &= a^{\log_a b / \log_a b} \\ &= (a^{\log_a b})^{1/\log_a b} \\ &= b^{1/\log_a b} \\ \log_b a &= \frac{1}{\log_a b} \end{aligned}$$

Some Properties of Logarithm
(continued)

$$\log_b x = \log_b (y^{\log_y x})$$
$$= \log_y x \log_b y$$
$$\log_b x = \frac{\log_n x}{\log_n b}$$
$$a^x = b^{x/\log_a b}$$
$$= b^{x \log_b a}.$$

Basic Calculus

導數的定義	$f'(x)=\frac{d}{dx}f(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$
冪法則	$\frac{d}{dx}x^n = nx^{n-1}$
函數和差之導數	$\frac{d}{dx}[f(x)\pm g(x)]=\frac{d}{dx}f(x)\pm\frac{d}{dx}g(x)$
積法則	$\frac{d}{dx}[f(x)g(x)]=f(x)\frac{d}{dx}g(x)+g(x)\frac{d}{dx}f(x)$
商法則	$\frac{d}{dx}\frac{f(x)}{g(x)}=\frac{g(x)\frac{d}{dx}f(x)-f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$

鏈式法則

$$\frac{dy}{dx}=\frac{dy}{du}\bullet\frac{du}{dx}$$
$$\frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}$$
$$\frac{d}{dx}y^n = ny^{n-1}\frac{dy}{dx}$$

對數的導數

$$\frac{d}{dx}\log_a x=\frac{\log_a e}{x}$$
$$\frac{d}{dx}\ln x=\frac{1}{x}$$

指數函數的導數

$$\frac{d}{dx}n^x = n^x \ln n$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}u^v = vu^{v-1}\frac{du}{dx} + u^v \ln u \frac{dv}{dx}$$

Determine the Function

- Assume the function is $Y=f(X)=a_0+a_1X+a_2x^2+\cdots+a_mx^m$
- We have a set of points (x_1,y_1), (x_2,y_2), $\cdots$ (x_n,y_n)
- Sum of offsets is defined as
- $R^2=\Sigma[y_i-f(x_i)]^2$, $1<=i<=n$, note a_0, a_1, $\cdots$, a_m are unknown and will be determined through least square method
- The idea is to minimize R^2, i.e., $\frac{\partial(R^2)}{\partial a_i}=0$

Linear Regression

- When m is 1 in $Y=f(X)=a_0+a_1X+a_2x^2+\cdots+a_mx^m$, we call this procedure linear regression
- $Y=f(X)=a_0+a_1X$,
- $R^2=\Sigma[y_i-f(x_i)]^2= \Sigma[y_i-(a_0+a_1x_i)]^2$, $1<=i<=n$

$$\frac{\partial(R^2)}{\partial a_i}=0$$

Linear Regression (Continued)

$$R^2 = \sum_{i=1}^{n}[y_i-(a_0+a_1x_i)]^2$$

$$\frac{\partial(R^2)}{\partial a_0} = -2\sum_{i=1}^{n}[y_i-(a_0+a_1x_i)] = 0$$

$$\frac{\partial(R^2)}{\partial a_1} = -2\sum_{i=1}^{n}[y_i-(a_0+a_1x_i)]x_i = 0$$

$$na_0 + a_1\sum_{i=1}^{n}x_i = \sum_{i=1}^{n}y_i$$

$$a_0\sum_{i=1}^{n}x_i + a_1\sum_{i=1}^{n}x_i^2 = \sum_{i=1}^{n}x_iy_i$$

Linear Regression (Continued)

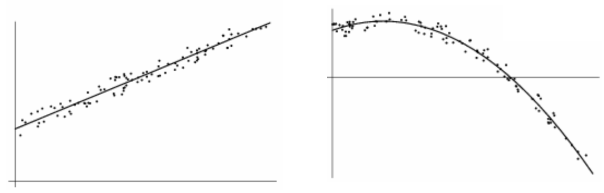
$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{bmatrix}^{-1}\begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2}\begin{bmatrix} \sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i y_i \\ n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i \end{bmatrix}$$

Least Square Method

- Find the best fitting curve to a given set of points by minimizing the sum of the squares of the offsets of the points from the curve.



Residuals (offsets)